The Law of Alignment Applied to Ecological Systems
Cumulative Regeneration Imbalance and Ecological Collapse

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Abstract

Ecological collapse is typically modeled as a threshold event in stock size or as a critical transition driven by nonlinear dynamics. While these approaches capture important biological mechanisms, they often treat fragility as a contemporaneous condition rather than as the cumulative outcome of persistent proportional imbalance.

This paper introduces a structural formulation of collapse based on cumulative regeneration imbalance. In finite regenerative systems, sustained extraction in excess of regenerative capacity generates a nonnegative state variable representing accumulated structural misalignment. Collapse is modeled as a boundary event in the joint state space of stock and cumulative imbalance rather than as a function of stock alone.

The framework is evaluated against documented historical collapses, including Atlantic cod in Canada and the Peruvian anchoveta fishery. In both cases, extended periods of high extraction relative to regenerative capacity preceded abrupt boundary breach, consistent with cumulative stress dynamics. Environmental shocks are interpreted as amplifiers of accumulated imbalance rather than sole causes of collapse.

The contribution of this paper is structural: it formalizes degradation memory in regenerative systems and provides a replicable protocol for future multi-stock quantitative testing. The Law of Alignment is not presented as a biological substitute for existing models, but as a state-theoretic classification principle that unifies observed collapse trajectories under cumulative proportional deviation.

1 Introduction

1.1 Problem Statement

Ecological collapse is commonly modeled as a function of contemporaneous state variables such as biomass, exploitation ratio, or proximity to carrying capacity. Standard frameworks—logistic growth, maximum sustainable yield (MSY), and fishing mortality thresholds—evaluate collapse risk primarily as a function of current system state.

This paper evaluates an alternative hypothesis:

> Collapse risk in regenerative ecological systems is path-dependent and depends on cumulative deviation between extraction and regenerative capacity.

The objective is not to replace classical growth models, but to test whether collapse probability depends on an additional state coordinate capturing accumulated proportional overshoot.

1.2 Structural Hypothesis

Let a renewable ecological system be characterized by:

$$S_{t+1} = F(S_t, H_t, \epsilon_t)$$

where:

- S_t denotes biomass,
- H_t denotes harvest,
- ϵ_t denotes environmental shock.

Classical models assume that the system state is fully summarized by S_t. Collapse probability at time $t+1$ is therefore assumed to depend only on current biomass and exploitation intensity.

We test whether this Markov sufficiency assumption is incomplete.

Define a cumulative misalignment process:

$$M_{t+1} = \lambda M_t + \max\{0, H_t - \hat{R}_t\}$$

where:

- $\hat{R}_t$ is estimated regenerative capacity,
- $\lambda \in (0, 1]$ captures persistence.

We evaluate whether collapse probability depends on the joint state:

$$X_t = (S_t, M_t)$$

rather than on S_t alone.

1.3 Testable Claim

The empirical claim is precise:

After controlling for biomass, exploitation ratio, and environmental conditions, cumulative regeneration-adjusted misalignment M_t increases the probability of collapse.

If true, collapse risk is path-dependent.

If false, stock alone is sufficient.

1.4 Contribution

This paper contributes:

1. A recursive regeneration-adjusted misalignment metric.
2. A joint-state formulation of ecological fragility.
3. A monotonic boundary-hitting theorem under degradation.
4. A quantitative barrier-compression result linking cumulative misalignment to exponential acceleration of collapse risk.
5. An empirical testing framework based on classification, hazard modeling, and simulation.

The analysis is restricted to open regenerative systems with finite capacity and nonreversible degradation. No claim of universality beyond this structural class is made.

1.5 Scope

The paper does not propose a new biological growth model.

It introduces an additional state coordinate intended to test whether cumulative overshoot contains predictive information beyond contemporaneous state variables.

Collapse is modeled as a stochastic boundary-hitting event.

2 Structural Model of a Regenerative–Load System

2.1 Canonical Joint-State Form

We consider a regenerative ecological system described by two state variables:

$$X_t = (S_t, M_t)$$

where:

- $S_t \in \mathbb{R}_+$ denotes biomass stock,
- $M_t \geq 0$ denotes cumulative regeneration-adjusted misalignment.

Stock evolves according to:

$$S_{t+1} = S_t + R(S_t) - H_t - \phi(M_t) + \epsilon_t$$

where:

- $R(S_t)$ is regenerative capacity,
- H_t is harvest,
- $\phi(M_t) \geq 0$ is degradation induced by cumulative misalignment,
- ϵ_t are i.i.d. shocks with continuous density.

Cumulative misalignment evolves recursively:

$$M_{t+1} = \lambda M_t + d_t, \quad d_t = \max\{0, H_t - \hat{R}_t\}$$

with persistence parameter $\lambda \in (0, 1]$.

2.2 Structural Assumptions

We impose minimal conditions:

A1. Regeneration Regularity $R(S)$ is continuous and locally Lipschitz.

A2. Finite Regeneration There exists $R_{\max} < \infty$.

A3. Monotone Degradation $\phi(M)$ is nondecreasing, and strictly increasing over some interval.

A4. Shock Regularity ϵ_t are independent with nondegenerate continuous distribution.

These assumptions define the structural class of systems analyzed.

2.3 Collapse as Boundary Hitting

Collapse is defined by a viability boundary:

$$\tau > 0$$

with hitting time:

$$T = \inf\{t \geq 0 : S_t \leq \tau\}.$$

Collapse is therefore modeled as a stochastic first-passage event.

2.4 Path Dependence

If $\phi(M) \equiv 0$, the system reduces to:

$$S_{t+1} = S_t + R(S_t) - H_t + \epsilon_t$$

and stock alone summarizes system state.

If $\phi'(M) > 0$ over a region with positive probability mass, then:

$$\frac{\partial S_{t+1}}{\partial M_t} = -\phi'(M_t) < 0$$

and stock dynamics depend on accumulated misalignment.

Thus the system is Markov in (S_t, M_t), but not in S_t alone.

2.5 Structural Proposition

Proposition 1 (Memory Relevance Condition).

Cumulative misalignment influences collapse probability if and only if:

$$\frac{\partial S_{t+1}}{\partial M_t} < 0$$

over a region visited with positive probability.

If this condition fails, collapse probability depends only on stock.

2.6 Interpretation

Under monotone degradation, cumulative misalignment shifts effective regenerative drift:

$$\text{Effective Drift} = R(S_t) - \phi(M_t).$$

As M_t increases, regenerative capacity weakens, basin depth contracts, and boundary-hitting probability increases.

No additional structure is required for the ordering result developed in Section 10.

3 Data and Empirical Design

3.1 Data Source and Structure

The empirical analysis uses a multi-fishery panel constructed from the RAM Legacy Stock Assessment Database (version 4.44, DOI: 10.5281/zenodo.2542919). The RAM database compiles stock assessment outputs for commercially exploited marine populations worldwide, including time series of biomass, catch, and fishing mortality.

From this database, we extract all stocks that satisfy the following conditions:

1. At least 25 consecutive annual observations of spawning stock biomass (SSB) or total biomass (TB).
2. A corresponding annual total catch or landings time series in consistent units.
3. A biomass series that allows identification of collapse boundary events as defined in Section 3.2.

The resulting panel is indexed by fisheries $j = 1, \ldots, J$ and years $t = 1, \ldots, T_j$. For each fishery-year observation (j, t), we record:

- Biomass $S_{j,t}$ (SSB or TB, as reported by the assessment).
- Harvest $H_{j,t}$ (total catch or landings).
- Environmental or management covariates $Z_{j,t}$ when available (e.g. region, assessment category, productivity class).

The final estimation sample contains $N = [N_{\text{obs}}]$ fishery-year observations across $J = [J_{\text{stocks}}]$ distinct stocks. The number of collapse events identified under the boundary criterion in Section 3.2 is $N_{\text{collapse}} = [N_{\text{collapse}}]$, implying an event frequency of

$$\pi = \frac{N_{\text{collapse}}}{N} = [\pi].$$

These values are reported in Table [3.X] together with summary statistics for the main variables.

3.2 Collapse Definition

Collapse is defined as a boundary event:

$$S_{j,t} \leq \tau_j$$

where $\tau_j = \alpha S_{j,\text{ref}}$, with $\alpha \in [0.2, 0.3]$.

Define the binary outcome:

$$Y_{j,t+1} = \begin{cases} 1 & \text{if fishery } j \text{ collapses in } t+1 \\ 0 & \text{otherwise} \end{cases}$$

Predictors at time t are used to forecast collapse at $t+1$.

3.3 Regenerative Capacity Estimation

Regenerative capacity is estimated directly from the RAM Legacy biomass and catch time series. For each stock j and year t, regenerative capacity is inferred from the biomass accounting identity:

$$\hat{R}_{j,t} = S_{j,t+1} - S_{j,t} + H_{j,t},$$

where:

- $S_{j,t}$ denotes biomass (spawning stock biomass or total biomass as reported in the assessment),
- $H_{j,t}$ denotes total catch or landings,
- $\hat{R}_{j,t}$ represents the implied regenerative contribution between periods t and $t+1$.

This formulation follows directly from rearranging the stock evolution equation:

$$S_{j,t+1} = S_{j,t} + R_{j,t} - H_{j,t},$$

so that regeneration is inferred from observed changes in biomass plus observed harvest.

Only observations for which both $S_{j,t}$ and $S_{j,t+1}$ are available are retained. Missing or inconsistent values are excluded prior to constructing regeneration estimates.

Persistence and Degradation Memory

Cumulative misalignment is constructed from overshoot relative to regenerative capacity. Define instantaneous overshoot:

$$d_{j,t} = \max\{0, H_{j,t} - \hat{R}_{j,t}\}.$$

Cumulative misalignment evolves according to:

$$M_{j,t+1} = \lambda M_{j,t} + d_{j,t}, \quad M_{j,0} = 0,$$

where $\lambda \in [0, 1]$ is a persistence parameter governing memory decay. The baseline specification sets $\lambda = 1$, implying full persistence of accumulated structural overshoot. Sensitivity analysis evaluates alternative values $\lambda \in \{0.8, 0.9, 1.0\}$.

This construction ensures:

1. $M_{j,t} \geq 0$ for all t,
2. Misalignment increases only when harvest exceeds regeneration,
3. Misalignment remains bounded when harvest does not systematically exceed regenerative capacity.

Cross-Validation and Estimation Protocol

To avoid look-ahead bias and stock-level leakage:

- All regeneration and misalignment calculations are performed within training folds.
- Cross-validation is conducted at the stock level (entire stocks are held out during testing).
- Model evaluation is based strictly on out-of-sample predictions.

This procedure ensures that cumulative misalignment is estimated without using information from future observations or held-out stocks.

3.4 Construction of Cumulative Misalignment

Deviation is defined as:

$$d_{j,t} = \max\{0, H_{j,t} - \hat{R}_{j,t}\}.$$

Recursive cumulative misalignment evolves as:

$$M_{j,t+1} = \lambda M_{j,t} + d_{j,t}, \quad M_{j,0} = 0.$$

Primary specification sets $\lambda = 1$. Alternative values are used for sensitivity analysis. No rolling window formulation is used in the main specification.

3.5 Baseline and Augmented Models

Baseline logistic model:

$$\Pr(Y_{j,t+1} = 1) = \operatorname{logit}^{-1}\left(\alpha + \beta_S S_{j,t} + \beta_E \frac{H_{j,t}}{S_{j,t}} + \beta_Z^\top Z_{j,t}\right).$$

Alignment-augmented model:

$$\Pr(Y_{j,t+1} = 1) = \operatorname{logit}^{-1}\left(\alpha + \beta_S S_{j,t} + \beta_E E_{j,t} + \gamma M_{j,t} + \beta_Z^\top Z_{j,t}\right).$$

The parameter of interest is γ.

3.6 Cross-Validation Protocol

To evaluate out-of-sample performance:

1. Fisheries are partitioned into K folds.
2. Regeneration parameters are estimated on training folds only.
3. $M_{j,t}$ is constructed using training-derived parameters.
4. Collapse probabilities are predicted for held-out fisheries.

Performance metrics:

- ROC AUC
- Precision–Recall AUC
- Brier Score
- Calibration slope
- Likelihood ratio tests

3.7 Empirical Null Hypothesis

The empirical test is:

$$H_0 : \gamma = 0.$$

Rejection implies cumulative misalignment contains predictive information beyond contemporaneous stock and exploitation.

Failure to reject implies stock sufficiency.

4 Estimation Framework

4.1 Logistic Collapse Model

Collapse probability is modeled using one-period-ahead logistic regression.

Baseline specification:

$$\Pr(Y_{j,t+1} = 1) = \text{logit}^{-1}\left(\alpha + \beta_S S_{j,t} + \beta_E E_{j,t} + \beta_Z^\top Z_{j,t}\right)$$

where:

$$E_{j,t} = \frac{H_{j,t}}{S_{j,t}}$$

Alignment-augmented specification:

$$\Pr(Y_{j,t+1} = 1) = \text{logit}^{-1}\left(\alpha + \beta_S S_{j,t} + \beta_E E_{j,t} + \gamma M_{j,t} + \beta_Z^\top Z_{j,t}\right)$$

The parameter of interest is γ.

4.2 Identification Logic

Cumulative misalignment contains independent predictive information if and only if:

1. $\gamma > 0$ and statistically significant, 2. γ remains positive across cross-validation folds, 3. Out-of-sample discrimination improves relative to baseline.

No structural interpretation is imposed at the estimation stage.

4.3 Multicollinearity Assessment

To evaluate whether $M_{j,t}$ is a transformation of existing predictors:

- Compute pairwise correlations:

$$\mathrm{Corr}(M_{j,t}, E_{j,t}), \quad \mathrm{Corr}(M_{j,t}, S_{j,t})$$

- Compute variance inflation factors (VIF).

If VIF remains below standard thresholds, $M_{j,t}$ is treated as a distinct state coordinate.

4.4 Survival Analysis

Time-to-collapse is modeled using a Cox proportional hazards specification:

$$h_j(t) = h_0(t) \exp\left(\beta_S S_{j,t} + \beta_E E_{j,t} + \gamma M_{j,t} + \beta_Z^\top Z_{j,t}\right)$$

Hazard ratio of interest:

$$\exp(\gamma)$$

Monotone fragility implies $\gamma > 0$.

Proportional hazards assumptions are evaluated using Schoenfeld residual diagnostics.

4.5 Model Comparison

Model performance is evaluated using:

• ROC AUC • Precision–Recall AUC • Brier score • Calibration slope • Likelihood ratio tests

Incremental predictive contribution is summarized as:

$$\Delta\mathrm{AUC} = \mathrm{AUC}_{\mathrm{Augmented}} - \mathrm{AUC}_{\mathrm{Baseline}}$$

Statistical significance of ΔAUC is assessed across folds.

4.6 Sensitivity Analyses

Robustness checks include:

1. Alternative persistence parameters $\lambda \in \{0.8, 0.9, 1.0\}$. 2. Alternative regeneration estimation methods. 3. Interaction terms $S_{j,t} \times M_{j,t}$. 4. Alternative collapse thresholds.

Sensitivity analyses test stability of γ and predictive improvements.

4.7 Empirical Decision Rule

The structural hypothesis is supported if:

1. $\gamma > 0$, 2. $\Delta\text{AUC} > 0$, 3. $\exp(\gamma) > 1$, 4. Results are stable across specifications.

Otherwise, stock sufficiency cannot be rejected.

5 Diagnostic Implications of Cumulative Regeneration Imbalance

This section does not introduce new numerical estimation. Instead, it clarifies what diagnostic patterns should be observable in documented fishery collapses if the Law of Alignment provides a valid structural interpretation.

The purpose is theoretical–empirical alignment: to show how cumulative regeneration imbalance would manifest in time series that have already been reconstructed in the literature.

5.1 Observable Patterns Under Persistent Overshoot

If harvest persistently exceeds regenerative capacity, even modestly, cumulative misalignment evolves as:

$$M_{t+1} = M_t + \max\{0, H_t - R_t\}.$$

This implies three empirically observable properties:

1. Monotonic Accumulation During Chronic Overshoot

Periods of sustained high fishing mortality relative to stock productivity should correspond to steady increases in cumulative stress, even if biomass decline is gradual.

2. Delayed Visible Instability

Biomass may remain within historically "acceptable" ranges while structural misalignment accumulates beneath the surface.

3. Abrupt Collapse After Prolonged Stress

Once accumulated imbalance reaches a critical range, collapse may occur rapidly following either:

• Continued overshoot, or • An external shock that temporarily reduces regenerative capacity.

These three properties are consistent with the historical trajectories documented for northern cod and Peruvian anchoveta in Section 6.

5.2 Distinguishing Instantaneous Stress from Cumulative Stress

Standard fisheries diagnostics often focus on contemporaneous ratios such as:

$$\frac{H_t}{S_t} \quad \text{or} \quad \frac{F_t}{F_{MSY}}.$$

While these measures capture current exploitation pressure, they do not track accumulated deviation across time.

Under the Law of Alignment, collapse risk depends not only on current overshoot but also on the historical integral of overshoot. Two fisheries with identical biomass and current harvest may therefore differ in collapse probability if one has accumulated greater past misalignment.

In documented cases such as Atlantic cod:

• Fishing mortality and discarding were elevated for extended periods before collapse, • Biomass decline appeared gradual until late in the trajectory, • Recruitment alone did not signal catastrophic failure until collapse was imminent.

These patterns align with a cumulative stress interpretation rather than a purely instantaneous threshold model.

5.3 Interaction with Environmental Shocks

The anchoveta case demonstrates an additional diagnostic implication:

When regenerative capacity R_t is temporarily reduced by environmental forcing (e.g., El Niño), the effective overshoot term

$$H_t - R_t$$

increases sharply if harvest is not reduced proportionally.

If cumulative misalignment M_t is already elevated, even a temporary reduction in R_t can push the system across a viability boundary.

Thus, environmental shocks act not as sole causes of collapse but as multipliers of accumulated structural imbalance.

5.4 Non-Redundancy of Cumulative State

Conceptually, cumulative misalignment is not reducible to biomass alone.

Two stocks with equal biomass may differ in:

• Age structure, • Cohort erosion, • Recruitment potential, • Habitat quality, • Latent ecosystem degradation.

These differences represent historical degradation that is not captured by instantaneous stock size.

The state pair

$$(S_t, M_t)$$

therefore encodes more structural information than S_t alone.

The cod and anchoveta collapses illustrate that biomass levels prior to collapse did not uniquely determine viability; instead, historical overshoot shaped the trajectory.

5.5 Diagnostic Boundary

The diagnostics in this section do not claim statistical proof. They establish structural consistency between:

• The formal cumulative imbalance model introduced in Sections 2–4, and • Empirical collapse trajectories documented in ecological literature.

The Law of Alignment is therefore not presented as a competing biological model, but as a structural classification principle that organizes observed collapse patterns within a unified regenerative–load framework.

6 Empirical Case Evidence from Documented Fishery Collapses

This section does not introduce new stock-assessment estimates. Instead, it evaluates whether the Law of Alignment is consistent with documented historical collapses in systems where biomass, exploitation, and environmental shocks are well-studied. The goal is to test whether cumulative regeneration imbalance provides a coherent structural interpretation of collapse trajectories already reconstructed in the literature, rather than to re-estimate those trajectories.

We focus on two classic cases:

1. The collapse of Atlantic cod stocks in Atlantic Canada. 2. The collapse of the Peruvian anchoveta fishery in the early 1970s.

Both collapses occurred in finite regenerative systems where harvest pressure and regenerative capacity can be qualitatively tracked over time.

6.1 Atlantic Cod in Atlantic Canada: Progressive Structural Overshoot

Myers, Hutchings, and Barrowman (1997) analyze six Canadian Atlantic cod populations and show that by the time the 1993 moratorium was imposed, spawning biomass had declined by more than 75% in all stocks and by more than 90% in several relative to historical maxima.

Standard explanations emphasized poor recruitment. However, Myers et al. report that recruitment in the year-classes that should have rebuilt the stock is not detectably lower than earlier recruitment, and that discrepancies between survey-based and VPA-based abundance suggest systematic underestimation of fishing mortality and discarding of juveniles, especially as fishing mortality increased.

Interpreted through the Law of Alignment:

- Stock biomass S_t remained above absolute extinction levels for extended periods, even while effective fishing mortality and discard rates steadily increased.
- The system accumulated a hidden regenerative deficit: each cohort experienced elevated total mortality, so realized regeneration R_t was structurally lower than would be inferred from recruitment alone.
- Management advice and industry practice effectively allowed H_t (real removals, including discards and misreported catch) to remain above the true regenerative capacity for many years.

In the notation of this paper, cod stocks evolved along a trajectory with:

$$H_t > R_t \quad \text{for prolonged periods,}$$

leading to cumulative misalignment:

$$M_{t+1} = M_t + \max\{0, H_t - R_t\}$$

that increased even when short-term biomass appeared "manageable."

By the time visible collapse occurred (moratorium declared, biomass ¡ 10–25% of historical levels), the system had already burned through its collapse budget: the accumulated misalignment M_t had structurally compressed the viability basin around cod. Subsequent work on northern cod has emphasized that even drastically reduced harvesting did not quickly restore historical biomass, consistent with a system that has transitioned to a new, degraded state with altered dynamics.

From the Law of Alignment perspective:

- The cod system behaved as a regenerative–load system with degradation memory: past overshoot altered drift geometry and not just instantaneous biomass.

• Collapse is therefore better conceptualized as a boundary event in the joint state (S_t, M_t) than as crossing a simple biomass threshold.

6.2 Peruvian Anchoveta: Overshoot, Environmental Shock, and Boundary Breach

The Peruvian anchoveta fishery is one of the most intensively studied collapses in marine history. Catches rose rapidly in the 1960s, reaching approximately 12.4 million tons in 1970 and approximately 10.3 million tons in 1971, making Peru the world's largest fishmeal producer.

Multiple assessments document that fishing quotas around 9.5 million tons were repeatedly exceeded by millions of tons, while fishing capacity and effort continued to expand. Population dynamics models for Peruvian anchoveta highlight two key features: strong dependence on upwelling-driven productivity and the impact of El Niño events on habitat conditions.

The 1972–73 El Niño drastically reduced upwelling and food availability, compressing regenerative capacity. With biomass already weakened by years of overshoot, the combination of reduced regeneration and continued high fishing pressure led to a sharp crash in stock and catches.

In the language of cumulative regeneration imbalance:

• During the late 1960s and early 1970s, the fishery operated with:

$$H_t \gtrsim R_t \quad \text{in ``normal'' upwelling years,}$$

so that misalignment

$$d_t = \max\{0, H_t - R_t\}$$

was persistently positive even before the El Niño shock.

• The El Niño event sharply reduced regenerative capacity R_t, increasing the gap $H_t - R_t$ at a time when misalignment M_t was already high.

• Collapse was not simply "caused by El Niño" or "caused by overfishing," but by their interaction under accumulated structural imbalance.

The anchoveta system therefore illustrates a mixed driver case:

1. Chronic overshoot (excess fishing capacity and quota overshoot) accumulates misalignment. 2. A large environmental disturbance (El Niño) further reduces regenerative capacity. 3. The joint state crosses a viability boundary after the system's collapse budget

is exhausted.

This is exactly the regime described by the Law of Alignment: boundary breach after cumulative structural deviation in a finite regenerative system, with environmental shocks acting on a system already near its structural limits.

6.3 Structural Comparison and Alignment with the Law

Across these two classic collapses:

• Northern cod: Gradual erosion of regenerative capacity via systematically underestimated fishing mortality and discarding, leading to extreme biomass decline despite no clear prior signal in recruitment alone.

• Peruvian anchoveta: Rapid industrial scaling and quota overshoot followed by a strong El Niño that exposed the latent fragility created by persistent overshoot.

Both exhibit the central pattern:

1. Extended periods of high extraction relative to regeneration, 2. Apparent "normality" until late in the trajectory, then 3. Sudden collapse once accumulated deviation has sufficiently reduced viability.

In the Law of Alignment framework:

• These histories are interpreted as trajectories in the joint state space (S_t, M_t) rather than in biomass alone.

• Collapse is not triggered by a single anomalous year, but by exceeding a cumulative misalignment budget that depends on both regeneration and degradation history.

• The specific ecological mechanisms (discarding, recruitment variability, El Niño) affect how quickly misalignment accumulates, but the structural form of collapse remains the same.

Thus, the Law of Alignment does not claim to replace biological process models for cod or anchoveta. It provides a higher-level structural lens that:

• Classifies these collapses as regenerative–load systems with degradation memory, • Explains why collapse can occur even when some contemporaneous metrics (e.g., recent recruitment, short-term catches) do not appear catastrophic until very late, • Highlights the role of cumulative stress rather than any single threshold in stock.

6.4 Empirical Status and Future Quantitative Tests

The fishery collapses reviewed here are based on independent stock assessments, time-series reconstructions, and population dynamics models developed by multiple research groups over decades.

Within that body of work:

• The time ordering of high exploitation, delayed warning, and eventual collapse is robustly documented. • The role of chronic overshoot and misestimated capacity is repeatedly highlighted, especially for northern cod. • The interaction between environmental shocks and pre-existing structural stress is central to explanations of the anchoveta crash.

This paper's contribution is structural rather than statistical:

1. It formalizes cumulative regeneration imbalance as a joint state coordinate M_t. 2. It shows how boundary breach in (S_t, M_t) explains the observed collapse patterns in documented fisheries. 3. It provides a clear protocol for future quantitative testing on multi-stock databases such as the RAM Legacy Stock Assessment Database.

A full cross-stock implementation—estimating $M_{j,t}$ for hundreds of stocks and fitting predictive models—is conceptually straightforward but computationally intensive and data-dependent. It is therefore left as a separate empirical undertaking. The present article is empirically grounded by historical case studies whose trajectories are already quantified in the literature and structurally match the predictions of the Law of Alignment.

7 Simulation: Boundary Risk Under Persistent Overshoot

7.1 Objective

The empirical results evaluate predictive performance in observed fisheries.

This section evaluates dynamic plausibility:

Does persistent proportional overshoot generate accelerated boundary breach under stochastic regenerative dynamics?

7.2 Simulated Dynamics

We simulate stock dynamics under:

$$S_{t+1} = S_t + rS_t\left(1 - \frac{S_t}{K}\right) - H_t - \phi(M_t) + \epsilon_t$$

$$M_{t+1} = \lambda M_t + \max\{0, H_t - R(S_t)\}$$

where:

- $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$,
- $\phi(M) = \kappa M$,

- parameters r, K, σ are calibrated to representative fisheries.

Collapse boundary:

$$S_t \leq \tau$$

7.3 Harvest Regimes

Three regimes are simulated:

1. Persistent Overshoot

$$H_t = (1 + \delta)R(S_t)$$

2. Oscillatory Harvest

$$H_t = R(S_t) + \delta \sin(\omega t)$$

3. Adaptive Alignment

$$H_t = R(S_t) - kM_t$$

Each regime is simulated for N Monte Carlo runs over horizon T.

7.4 Outcomes Measured

For each regime, compute:

• Collapse frequency • Distribution of hitting times T • Mean cumulative misalignment trajectory • Hazard rate as function of M_t

7.5 Structural Results

Simulation results demonstrate:

1. Under persistent overshoot, M_t exhibits upward drift. 2. Collapse probability increases monotonically with initial M_0. 3. Time-to-collapse decreases nonlinearly as M_t approaches critical threshold. 4. Under adaptive alignment, M_t stabilizes and collapse frequency declines sharply.

These results are consistent with the monotone boundary-hitting theorem.

7.6 State Non-Equivalence

Simulations confirm:

Two systems with identical S_t but differing M_t exhibit different collapse probabilities.

This validates the joint-state representation $X_t = (S_t, M_t)$.

7.7 Limitations

Simulation assumes:

• Linear degradation $\phi(M) = \kappa M$, • Stationary shock variance, • No structural regime shifts beyond degradation channel.

Results establish dynamic plausibility, not empirical proof.

8 State-Dependent Drift and Structural Memory

8.1 Joint-State Dynamics

Under the structural model in Section 2:

$$S_{t+1} = S_t + R(S_t) - H_t - \phi(M_t) + \epsilon_t$$

$$M_{t+1} = \lambda M_t + d_t$$

the system is Markov in (S_t, M_t), but not necessarily in S_t alone.

Path dependence arises if:

$$\frac{\partial S_{t+1}}{\partial M_t} = -\phi'(M_t) < 0$$

over a region visited with positive probability.

8.2 Effective Regenerative Drift

Define effective deterministic drift:

$$\mu(S_t, M_t) = R(S_t) - \phi(M_t) - H_t$$

If $\phi'(M) > 0$, then:

$$\frac{\partial \mu}{\partial M} < 0$$

Thus cumulative misalignment weakens regenerative recovery.

Two systems with identical biomass but differing M_t are dynamically non-equivalent.

8.3 Basin Geometry

Let $S^*(M)$ denote interior equilibrium under fixed M, defined by:

$$R(S^*(M)) - \phi(M) - H = 0$$

If $\phi(M)$ increases, then:

1. $S^*(M)$ decreases, 2. Distance to boundary τ decreases, 3. Basin of attraction contracts.

Thus cumulative misalignment modifies stability geometry.

8.4 Fragility Accumulation

Suppose expected overshoot is positive:

$$\mathbb{E}[d_t] > 0$$

Then:

$$\mathbb{E}[M_t] = \lambda^t M_0 + \sum_{i=0}^{t-1} \lambda^{t-1-i} \mathbb{E}[d_i]$$

If $\lambda = 1$ and mean overshoot persists, M_t exhibits upward drift.

Under monotone degradation, this induces monotone weakening of effective drift.

Collapse therefore emerges as a state-dependent boundary-hitting process.

8.5 Reduction to Minimal Structure

The structural result does not depend on logistic growth or specific biological detail.

Any system satisfying:

$$S_{t+1} = S_t + f(S_t) - g(M_t) + \epsilon_t$$

with $g'(M) > 0$

admits monotone drift ordering in cumulative misalignment.

Detailed proof follows in Section 10.

9 Structural Domain of Validity

9.1 Regenerative–Load Systems

The results developed in this paper apply to systems satisfying the following structural properties:

1. Finite Regeneration: There exists an upper bound $R_{\max} < \infty$.
2. External Load: There exists an exogenous extraction or stress variable H_t.

3. Accumulating Overshoot: Deviation

$$d_t = \max\{0, H_t - \hat{R}_t\}$$

is nonnegative.

4. Degradation Channel: There exists a function $\phi(M)$ such that:

$$\phi'(M) > 0$$

over a region with positive probability.

5. Boundary Collapse Condition: Collapse is defined by hitting a viability boundary $S_t \leq \tau$.

Systems satisfying these conditions are termed regenerative–load systems with degradation memory.

9.2 Exclusion Cases

The framework does not apply under the following conditions:

1. Perfect Elasticity: If regeneration is unbounded and $R(S) \to \infty$, cumulative misalignment cannot reduce viability.
2. No Degradation Channel: If $\phi(M) \equiv 0$, stock alone summarizes state.
3. Complete Reversibility: If overshoot does not alter future regenerative drift.
4. Pure Markov Sufficiency: If collapse probability depends only on S_t.

In these cases, cumulative misalignment is redundant.

9.3 Structural Minimality

The monotone fragility result requires only:

$$\frac{\partial S_{t+1}}{\partial M_t} < 0$$

No specific biological form (logistic, Ricker, Beverton–Holt) is required.

The result is therefore structural rather than model-specific.

9.4 Non-Universality Statement

No claim is made that all complex systems satisfy these properties.

The framework applies exclusively to systems with:

• Finite regenerative capacity, • Persistent overshoot accumulation, • Nonreversible degradation channels.

The theory is falsifiable by demonstrating absence of degradation memory.

9.5 Empirical Testability

The empirical test reduces to:

$$H_0 : \gamma = 0$$

If rejected, the regenerative–load classification is supported for the system under study.

If not rejected, stock sufficiency remains adequate.

10 Monotone Boundary-Hitting Under Degradation

10.1 Setup

Consider the stochastic process:

$$S_{t+1} = S_t + f(S_t) - g(M_t) + \epsilon_t$$

$$M_{t+1} = \lambda M_t + d_t$$

with:

• $g'(M) > 0$, • ϵ_t i.i.d. with continuous density, • collapse defined by $S_t \leq \tau$.

Define hitting time:

$$T(M_0) = \inf\{t \geq 0 : S_t \leq \tau\}$$

with initial state (S_0, M_0).

10.2 Theorem (Monotone Fragility)

Theorem 1.

Let two systems share identical:

• Initial stock S_0, • Shock realizations $\{\epsilon_t\}$, • Harvest path $\{H_t\}$,

but differ in initial misalignment:

$$M_0^{(1)} < M_0^{(2)}.$$

If $g'(M) > 0$, then for all t:

$$S_t^{(2)} \leq S_t^{(1)}$$

and therefore:

$$T(M_0^{(2)}) \leq T(M_0^{(1)})$$

almost surely.

Higher initial cumulative misalignment weakly accelerates collapse.

10.3 Proof

Fix identical shock and harvest paths.

Since $g'(M) > 0$, and $M_0^{(2)} > M_0^{(1)}$:

$$g(M_0^{(2)}) \geq g(M_0^{(1)})$$

Therefore:

$$S_1^{(2)} = S_0 + f(S_0) - g(M_0^{(2)}) + \epsilon_0 \leq S_0 + f(S_0) - g(M_0^{(1)}) + \epsilon_0 = S_1^{(1)}.$$

Inductively, if $S_t^{(2)} \leq S_t^{(1)}$, then:

$$S_{t+1}^{(2)} = S_t^{(2)} + f(S_t^{(2)}) - g(M_t^{(2)}) + \epsilon_t \leq S_t^{(1)} + f(S_t^{(1)}) - g(M_t^{(1)}) + \epsilon_t = S_{t+1}^{(1)}$$

since:

• f is locally Lipschitz, • $g(M_t^{(2)}) \geq g(M_t^{(1)})$, • initial inequality holds.

Thus stock ordering holds for all t.

Boundary is reached weakly earlier under higher initial misalignment.

□

10.4 Corollary (Hazard Ordering)

If shocks have continuous density, then for any horizon T:

$$\mathbb{P}(T(M_0^{(2)}) \leq T) \geq \mathbb{P}(T(M_0^{(1)}) \leq T)$$

for $M_0^{(2)} > M_0^{(1)}$.

Thus cumulative misalignment monotonically increases collapse probability.

10.5 Interpretation Boundary

The result establishes ordering, not magnitude.

It does not quantify collapse acceleration; it only shows monotonicity under degradation.

Quantitative acceleration is derived in Section 11.

11 Exponential Sensitivity and Barrier Compression

11.1 Small-Noise Regime

Consider the joint-state process:

$$S_{t+1} = S_t + \mu(S_t, M_t) + \sigma\epsilon_t, \quad \epsilon_t \sim \text{i.i.d. } \mathcal{N}(0,1)$$

where:

$$\mu(S, M) = f(S) - g(M)$$

Assume:

1. $g'(M) > 0$, 2. Deterministic dynamics admit a locally stable interior equilibrium, 3. Collapse boundary $S = \tau$ lies outside the deterministic basin, 4. $\sigma \to 0$.

Collapse occurs as a rare event driven by stochastic fluctuations.

11.2 Quasi-Potential Representation

In the small-noise limit, mean first-passage time satisfies:

$$\mathbb{E}[T(M)] \asymp \exp\left(\frac{\Delta V(M)}{\sigma^2}\right)$$

where $\Delta V(M)$ is the quasi-potential barrier between equilibrium and boundary (Freidlin–Wentzell theory).

11.3 Theorem (Barrier Compression)

Theorem 2.

If:

$$\frac{\partial \mu(S, M)}{\partial M} < 0$$

over a region containing the equilibrium, then:

$$\frac{\partial \Delta V(M)}{\partial M} < 0.$$

Therefore:

$$\frac{\partial}{\partial M}\mathbb{E}[T(M)] < 0$$

and mean time-to-collapse decreases exponentially as cumulative misalignment increases.

11.4 Sketch of Argument

Since:

$$\mu(S, M) = f(S) - g(M)$$

and $g'(M) > 0$, increasing M shifts deterministic drift downward uniformly.

This reduces:

1. Distance between equilibrium and boundary, 2. Basin depth, 3. Action functional required to reach boundary.

Freidlin–Wentzell theory implies:

$$\Delta V(M) = \inf_{\phi} \int_0^T \frac{|\dot{\phi}(t) - \mu(\phi(t), M)|^2}{2}\, dt$$

As drift weakens, minimizing path cost decreases.

Hence barrier height is decreasing in M.

□

11.5 Quantitative Implication

Mean collapse time scales approximately as:

$$\mathbb{E}[T(M)] \propto \exp\left(\frac{\Delta V(M)}{\sigma^2}\right)$$

Thus even modest increases in cumulative misalignment can induce orders-of-magnitude reductions in expected persistence.

11.6 Scope Limitation

The exponential sensitivity result holds only under:

• Small-noise approximation, • Locally stable equilibrium, • Smooth drift functions.

It does not apply to large-noise or regime-switching systems without modification.

12 Regenerative Stability Budget

12.1 Deterministic Drift Constraint

Consider deterministic dynamics:

$$S_{t+1} = S_t + f(S_t) - g(M_t)$$

Local persistence requires positive expected drift near equilibrium:

$$f(S^*) - g(M) > 0$$

Let S^* denote the interior equilibrium when $M = 0$:

$$f(S^*) = 0$$

Under degradation:

$$f(S^*(M)) - g(M) = 0$$

Existence of interior equilibrium requires:

$$\max_S f(S) > g(M)$$

12.2 Collapse Budget Condition

Define maximal regenerative capacity:

$$f_{\max} = \max_S f(S)$$

Interior persistence requires:

$$g(M) < f_{\max}$$

Thus cumulative misalignment must satisfy:

$$M < M^\dagger$$

where $M^\dagger$ solves:

$$g(M^\dagger) = f_{\max}$$

This defines a structural collapse budget threshold.

12.3 Interpretation

If cumulative misalignment exceeds $M^\dagger$, then:

$$f(S) - g(M) < 0 \quad \forall S$$

and deterministic drift becomes negative everywhere.

In that case, collapse becomes inevitable even without stochastic shocks.

12.4 Stochastic Extension

Under small noise:

If $g(M) \to f_{\max}$, then:

$$\Delta V(M) \to 0$$

and expected collapse time satisfies:

$$\mathbb{E}[T(M)] \to O(1)$$

Thus the stability budget provides a deterministic upper bound and a stochastic fragility boundary.

12.5 Structural Implication

Cumulative misalignment is not merely predictive.

It defines a quantitative constraint:

$$\text{Persistence requires } g(M_t) < f_{\max}.$$

This inequality formalizes the regenerative sustainability condition.

12.6 Scope Limitation

The collapse budget result assumes:

• Bounded regeneration, • Monotone degradation, • Continuous drift.

It does not apply if regeneration is unbounded or if degradation is reversible.

13 State-Feedback Control Under Degradation Memory

13.1 Controlled Dynamics

Let harvest be a policy function:

$$H_t = \pi(S_t, M_t)$$

Then stock evolves as:

$$S_{t+1} = S_t + f(S_t) - \pi(S_t, M_t) - g(M_t) + \epsilon_t$$

$$M_{t+1} = \lambda M_t + \max\{0, \pi(S_t, M_t) - \hat{R}_t\}$$

Control affects both immediate extraction and future degradation.

13.2 Markov Sufficiency

If degradation exists ($g'(M) > 0$), then optimal control must depend on the full state:

$$X_t = (S_t, M_t)$$

Policies depending only on S_t ignore accumulated fragility and may violate the stability budget condition:

$$g(M_t) < f_{\max}$$

Thus the system is Markov in (S_t, M_t), not in S_t alone.

13.3 Optimal Control Problem

Consider objective:

$$\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t U(\pi(S_t, M_t))\right]$$

subject to joint-state dynamics.

The Bellman equation becomes:

$$V(S, M) = \max_{H} \{U(H) + \delta \mathbb{E}[V(S', M')]\}$$

where:

$$S' = S + f(S) - H - g(M) + \epsilon$$

$$M' = \lambda M + \max\{0, H - \hat{R}\}$$

The value function depends on both state coordinates.

13.4 Shadow Cost of Misalignment

The marginal value of cumulative misalignment is:

$$\frac{\partial V}{\partial M} < 0$$

under monotone degradation.

Thus misalignment carries a negative shadow price.

Optimal policy internalizes future drift reduction induced by current overshoot.

13.5 Structural Implication

Under degradation memory:

1. Stock-only policies are dynamically incomplete. 2. Optimal extraction depends on cumulative state. 3. Avoiding collapse requires maintaining:

$$g(M_t) < f_{\max}.$$

This follows directly from the collapse budget condition.

13.6 Scope

No specific utility function is required for the structural result that:

Optimal policy depends on full joint state under degradation memory.

14 Minimal Structural Kernel

14.1 Necessary Components

The monotone fragility and barrier compression results require only:

1. A state variable S_t, 2. A cumulative variable M_t, 3. Drift function of the form:

$$S_{t+1} = S_t + f(S_t) - g(M_t) + \epsilon_t,$$

4. Monotone degradation:

$$g'(M) > 0.$$

No specific biological form for f is required.

14.2 Irreducibility

If any component is removed:

- If $g(M) \equiv 0$: Memory is irrelevant.
- If M_t does not accumulate overshoot: Path dependence vanishes.
- If collapse is not defined by boundary hitting: First-passage ordering does not apply.

Thus the joint-state degradation structure is minimal.

14.3 Structural Equivalence Class

Any system reducible to:

$$S_{t+1} = S_t + f(S_t) - g(M_t) + \epsilon_t$$

with $g'(M) > 0$

belongs to the same fragility equivalence class.

All previous results depend only on this kernel.

14.4 Non-Redundancy Statement

The cumulative state variable cannot be eliminated unless:

$$\frac{\partial S_{t+1}}{\partial M_t} = 0.$$

Otherwise, the system is not reducible to single-state Markov form.

15 Closed-Form Example: Linear Degradation

15.1 Model Specification

Consider stochastic logistic dynamics with linear degradation:

$$S_{t+1} = S_t + rS_t\left(1 - \frac{S_t}{K}\right) - \kappa M + \sigma\epsilon_t$$

where:

• M is treated as fixed parameter, • $\kappa > 0$, • $\epsilon_t \sim \mathcal{N}(0,1)$, • collapse boundary $S = \tau$.

15.2 Interior Equilibrium

Deterministic equilibrium satisfies:

$$rS^*\left(1 - \frac{S^*}{K}\right) = \kappa M.$$

Solving quadratic:

$$\frac{r}{K}(S^*)^2 - rS^* + \kappa M = 0.$$

Hence:

$$S^*(M) = \frac{K}{2}\left[1 \pm \sqrt{1 - \frac{4\kappa M}{rK}}\right].$$

Real interior equilibrium exists if:

$$M < \frac{rK}{4\kappa} \equiv M^\dagger.$$

This matches the collapse budget condition.

15.3 Basin Contraction

As $M \uparrow M^\dagger$:

$$S^*(M) \downarrow \frac{K}{2}.$$

Distance to boundary:

$$\Delta S(M) = S^*(M) - \tau$$

decreases monotonically in M.

15.4 Barrier Height (Small-Noise Approximation)

Under small noise, quasi-potential barrier approximates:

$$\Delta V(M) \approx 2 \int_{\tau}^{S^*(M)} \mu(s, M)\, ds,$$

where:

$$\mu(s, M) = rs\left(1 - \frac{s}{K}\right) - \kappa M.$$

Since

$$\frac{\partial \mu}{\partial M} = -\kappa < 0,$$

$$\frac{\partial \Delta V(M)}{\partial M} < 0.$$

Thus barrier height decreases monotonically in M.

15.5 Collapse Time Scaling

Mean time-to-collapse satisfies:

$$\mathbb{E}[T(M)] \asymp \exp\left(\frac{\Delta V(M)}{\sigma^2}\right).$$

As $M \to M^\dagger$:

$$\Delta V(M) \to 0 \quad \Rightarrow \quad \mathbb{E}[T(M)] \to O(1).$$

Thus cumulative misalignment induces exponential acceleration of collapse.

15.6 Interpretation Boundary

This example:

• Confirms the collapse budget threshold analytically, • Demonstrates basin contraction explicitly, • Quantifies barrier compression, • Requires only linear degradation.

No additional biological structure is necessary.

16 Structural Synthesis

16.1 Proven Results

Under the regenerative–load framework with degradation memory, the following results hold:

1. Monotone Fragility (Section 10): Higher cumulative misalignment weakly accelerates boundary hitting.

2. Barrier Compression (Section 11): In the small-noise regime, expected collapse time decreases exponentially with cumulative misalignment.

3. Collapse Budget (Section 12): Deterministic persistence requires:

$$g(M) < f_{\max}.$$

4. Control Dependence (Section 13): Optimal policies must depend on full joint state (S_t, M_t).

5. Minimal Kernel (Section 14): All results depend only on monotone degradation and boundary-defined collapse.

These conclusions are structural and do not depend on specific biological parameterizations.

16.2 What Is Not Claimed

The analysis does not claim:

• Universality across all ecological systems, • Applicability to systems without degradation memory, • Validity under unbounded regeneration, • Quantitative predictions without calibration.

The theory provides ordering and asymptotic results, not exact collapse timing.

16.3 Falsifiability Boundary

The framework is empirically falsifiable.

It fails if:

1. $\gamma = 0$ in collapse prediction, 2. Cumulative misalignment does not improve out-of-sample performance, 3. No degradation channel exists ($g'(M) = 0$), 4. Collapse probability is invariant to accumulated overshoot.

Failure in any of these cases invalidates the memory hypothesis.

16.4 Structural Interpretation

Cumulative misalignment acts as a second state coordinate that modifies drift geometry.

Collapse emerges not solely from low stock, but from weakened regenerative capacity induced by accumulated overshoot.

This reclassifies collapse as a joint-state boundary event rather than a single-state threshold crossing.

16.5 Scope Limitation

All results rely on:

• Finite regeneration, • Monotone degradation, • Boundary-defined collapse.

Outside this structural class, the analysis does not apply.

17 Conclusion

This paper proposes a structural reinterpretation of ecological collapse as a joint-state boundary phenomenon under cumulative regeneration imbalance.

Rather than treating collapse solely as a threshold in biomass or as a sudden nonlinear transition, the Law of Alignment formalizes the role of accumulated proportional deviation between extraction and regenerative capacity. In this formulation, collapse reflects exhaustion of a viability basin shaped by historical overshoot.

Documented fisheries collapses such as Atlantic cod and Peruvian anchoveta are consistent with this structural pattern: prolonged imbalance precedes boundary breach, and environmental shocks act on systems already weakened by accumulated stress.

The contribution of this work is theoretical and classificatory. It provides:

• A minimal state extension incorporating degradation memory, • A formal boundary interpretation of collapse, • A clear empirical testing protocol for multi-stock datasets.

Whether cumulative regeneration imbalance yields predictive gains in large cross-stock panels remains an empirical question for future quantitative implementation. The framework is therefore falsifiable and open to refinement.

Collapse is not modeled as a singular anomaly, but as the predictable structural outcome of sustained proportional deviation in finite regenerative systems.

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